Class- X Session- 2022-23

Subject- Mathematics (Basic)

Sample Question Paper - 3

with Solution

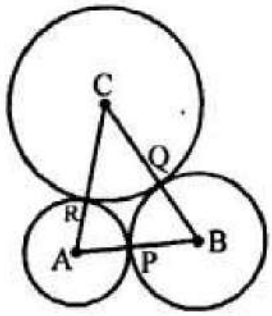
Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. In the given figure, three circles with centres A, B, C respectively touch each other externally. If AB = 5 cm, BC = 7 cm and CA = 6 cm, then the radius of the circle with centre A is **[1]**



a) 3 cm | b) 1.5 cm

c) 2 cm | d) 2.5 cm

2. The point (-3, 5) lies in the ________ quadrant **[1]**

a) 4th | b) 2nd

c) 3rd | d) 1st

3. If two different dice are rolled together, the probability of getting an even number **[1]**

a) $\frac{1}{2}$ | b) $\frac{1}{4}$

c) $\frac{1}{36}$ | d) $\frac{1}{6}$

4. The value of k so that the system of equations 3x - 4y - 7 = 0 and 6x - ky - 5 = 0 have a unique solution is **[1]**

a) $k \neq -8$

b) $k \neq 4$

c) $k \neq -4$

d) $k \neq 8$

5. The distance of the point (4, 7) from the y-axis is **[1]**

a) 11

b) 4

c) $\sqrt{6}5$

d) 7

6. A die is thrown once. The probability of getting a prime number is **[1]**

a) $\frac{1}{3}$

b) $\frac{1}{6}$

c) $\frac{1}{2}$

d) $\frac{2}{3}$

7. If A (2, 2), B (-4, - 4) and C (5, -8) are the vertices of a triangle, then the length of the median through vertex C is **[1]**

a) $\sqrt{113}$

b) $\sqrt{65}$

c) $\sqrt{85}$

d) $\sqrt{117}$

8. If an event cannot occur then its probability is **[1]**

a) $\frac{3}{4}$

b) $\frac{1}{2}$

c) 0

d) 1

9. A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 40 m, the total area of the canvas required in m^2 is **[1]**

a) 1760

b) 7920

c) 2640

d) 3960

10. The quadratic equation whose roots are $7 + \sqrt{3}$ and $7 - \sqrt{3}$ is **[1]**

a) $x^2 + 14x - 46 = 0$

b) $x^2 - 14x + 46 = 0$

c) $x^2 - 14x - 46 = 0$

d) $x^2 + 14x + 46 = 0$

11. $x^2 - 6x + 6 = 0$ have **[1]**

a) Real and Equal roots

b) Real roots

c) No Real roots

d) Real and Distinct roots

12. For every positive integer n, $n^2 - n$ is divisible by **[1]**

a) 6

b) 4

c) 2

d) 8

13. The ratio in which the line segment joining P(x_1, y_1) and Q(x_2, y_2) is divided by x-axis is [1]

a) $y_1 : y_2$
b) $-y_1 : y_2$
c) $-x_1 : x_2$
d) $x_1 : x_2$

14. If $\cos\theta = \frac{4}{5}$ then $\tan\theta = ?$ [1]

a) $\frac{3}{4}$
b) $\frac{5}{3}$
c) $\frac{4}{3}$
d) $\frac{3}{5}$

15. If the mode of the data: 16, 15, 17, 16, 15, x, 19, 17, 14 is 15, then x = [1]

a) 19
b) 15
c) 16
d) 17

16. The ratio between the height and the length of the shadow of a pole is $1:\sqrt{3}$, then the sun's altitude is [1]

a) 60°
b) 30°
c) 75°
d) 45°

17. The graphs of the equations 6x - 2y + 9 = 0 and 3x - y + 12 = 0 are two lines which are [1]

a) perpendicular to each other
b) parallel
c) coincident
d) intersecting exactly at one point

18. The product of two numbers is 1600 and their HCF is 5. The LCM of the numbers is [1]

a) 1600
b) 8000
c) 1605
d) 320

19. **Assertion (A):** If a line intersects sides AB and AC of a $\triangle$ABC at D and E respectively and is parallel to BC, then $\frac{AD}{AB} = \frac{AE}{AC}$ [1]
Reason (R): If a line is parallel to one side of a triangle then it divides the other two sides in the same ratio.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The HCF of two numbers is 5 and their product is 150, then their LCM is 30. [1]
Reason (R): For any two positive integers a and b, HCF (a, b) + LCM (a, b) = a × b.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. A piggy bank contains hundred 50 p coins, fifty ₹ 1 coins, twenty ₹ 2 coins and ten ₹ 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin **[2]**

i. will be a 50 p coin?

ii. will not be a ₹ 5 coin?

22. Find a quadratic polynomial of 4, 1 as the sum and product of its zeroes respectively. **[2]**

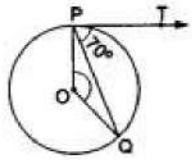
23. Determine the ratio in which the line 2x + y - 4 = 0 divides the line segment joining the points A(2, -2) and B(3, 7). **[2]**

24. Is the pair of linear equation consistent/ inconsistent? If consistent, obtain the solution graphically: x – y = 8; 3x – 3y = 16. **[2]**

OR

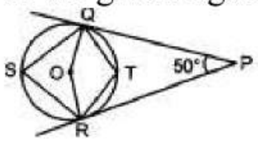
Find the value(s) of p and q in the pair of the equation: 2x + 3y = 7 and 2px + py = 28 – qy, if the pair of equations have infinitely many solutions.

25. If PT is a tangent to a circle with centre O and PQ is a chord of the circle such that $\angle QPT$= 70°, then find the measure of $\angle POQ$. **[2]**



OR

In the given figure, find ∠QSR.



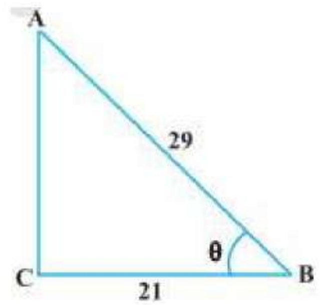
Section C

26. Solve the following pair of linear equations by the elimination method and the substitution method: 3x + 4y = 10 and 2x – 2y = 2. **[3]**

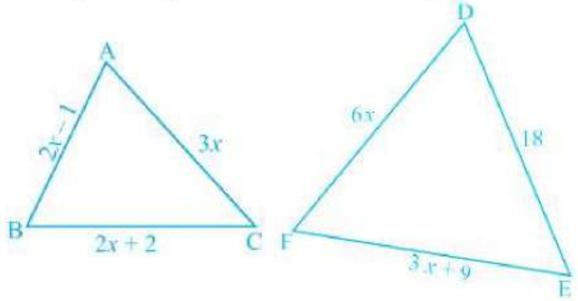
27. Consider $\triangle ACB$ right angled at C in which AB = 29 units, BC = 21 units and $\angle ABC = \theta$. Determine the values of **[3]**

i. $\cos^2 \theta + \sin^2 \theta$

ii. $\cos^2 \theta - \sin^2 \theta$



28. In the figure, if $\triangle ABC \sim \triangle DEF$ and their sides are of lengths (in cm) as marked along them, then find the lengths of the sides of each triangle. **[3]**



29. Prove that $\sqrt{5}$ is irrational. **[3]**

OR

Find the HCF and LCM of the following pairs of positive integers by applying the prime factorization method: 72, 90

30. A tower stands vertically on the ground. From a point on the ground 100 m away from the foot of the tower, the angle of elevation of the top of the tower is 45°. Find the height of the tower. **[3]**

31. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. **[3]**

OR

Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

Section D

32. O is the point of intersection of the diagonals AC and BD of a trapezium ABCD with AB || DC. Through O, a line segment PQ is drawn parallel to AB meeting AD in P and BC in Q. Prove that PO = QO. **[5]**

33. If the roots of the quadratic equation (x - a) (x - b) + (x - b)(x - c) + (x - c)(x - a) = 0 are equal. Then show that a = b = c **[5]**

OR

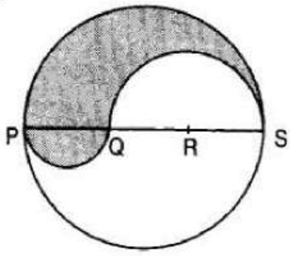
Find the values of k for which the equation $(3k+1)x^2+2(k+1)x+1$,has equal roots. Also find the roots.

34. The following table shows the ages of the patients admitted in a hospital during a year: **[5]**

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of patients	6	11	21	23	14	5

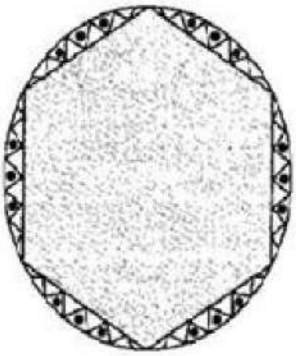
Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

35. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. Find the perimeter and area of the shaded region **[5]**



OR

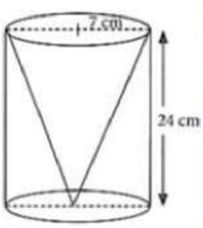
A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm ^{2}. (use $\sqrt{3}$ = 1.7)



Section E

36. **Read the text carefully and answer the questions:** **[4]**

One day Vinod was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Vinod's mind.



(i) Find the slant height of the conical cavity so formed?

(ii) Find the curved surface area of the conical cavity so formed?

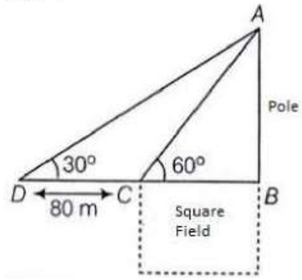
(iii) Find the external curved surface area of the cylinder?

OR

Find the ratio of curved surface area of cone to curved surface area of cylinder?

37. **Read the text carefully and answer the questions:** **[4]**

Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite to this corner is 60^{o}. When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30^{o}.



(i) Find the height of the pole too so that he can arrange a ladder accordingly to put an effigy on the pole.

(ii) Find the length of his square field so that he can buy material to do the fencing work accordingly.

(iii) Find the Distance from Farmer at position C and top of the pole?

OR

Find the Distance from Farmer at position D and top of the pole?

38. **Read the text carefully and answer the questions:** **[4]**

Suman is celebrating his birthday. He invited his friends. He bought a packet of toffees/candies which contains 360 candies. He arranges the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.

(i) Find the total number of rows of candies.

(ii) How many candies are placed in last row?

(iii) If Aditya decides to make 15 rows, then how many total candies will be placed by him with the same arrangement?

OR

Find the number of candies in 12th row.

Solution

Section A

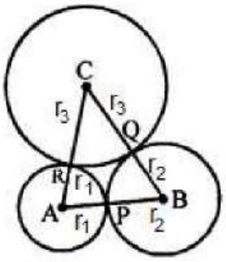
1. **(c)** 2 cm

   Explanation:

   In the given figure, three circles with centre A, B and C are drawn touching each other externally

   AB = 5 cm, BC = 7 cm and CA = 6 cm

   Let r_1, r_2, r_3 be the radii of three circles respectively

   $\therefore$AB = $r_1 + r_2$ = 5 cm ...(i)

   BC = $r_2 + r_3$ = 7 cm ...(ii)

   CA = $r_3 + r_1$ = 6 cm ....(iii)

   Adding, $2(r_1 + r_2 + r_3)$ = 18cm ---- (iv)

   Now, subtracting (ii) from(iv) respectively

   we get r_1 = 2 cm

   Hence, radius of the circle with centre A = 2 cm
2. **(b)** 2nd

   Explanation: Since x-coordinate is negative and y-coordinate is positive.

   Therefore, the point (-3, 5) lies in II quadrant.
3. **(b)** $\frac{1}{4}$

   Explanation: Rolling two different dice, Number of total events = 6 $\times$ 6 = 36

   Number of even number on both dice are {(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6) }= 9

   $\therefore\ Probability = \frac{9}{36} = \frac{1}{4}$
4. **(d)** k $\neq$ 8

   Explanation: Given: a_1 = 3, a_2 = 6, b_1 = -4, b_2 = -k, c_1 = -7 and c_2 = -5

   If there is a unique solution, then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

   $\Rightarrow \frac{3}{6} \neq \frac{-4}{-k}$

   $\Rightarrow$ -3k $\neq$ -4 $\times$ 6

   $\Rightarrow$ k $\neq$ 8
5. **(b)** 4

   Explanation: The distance of the point (4, 7) from y-a x is = 4
6. **(c)** $\frac{1}{2}$

   Explanation: Prime number on a die are 2, 3, 5

   $\therefore$ Probability of getting a prime number on the face of the die $= \frac{3}{6} = \frac{1}{2}$
7. **(c)** $\sqrt{85}$

   Explanation: Let mid point of A(2, 2), B(-4, -4) be whose coordinates will be

   $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2-4}{2}, \frac{2-4}{2}\right)$

or $\left(\frac{-2}{2}, \frac{-2}{2}\right) = (-1, -1)$

$\therefore$ Length of median CD

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5+1)^2 + (-8+1)^2}$

$= \sqrt{(6)^2 + (-7)^2} = \sqrt{36 + 49}$

$= \sqrt{85}$ units

8. **(c)** 0

Explanation: The event which cannot occur is said to be impossible event and probability of impossible event is zero.

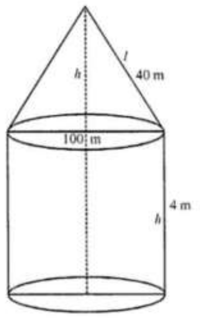
9. **(b)** 7920

Explanation: Diameter of tent = 105 m

Height of the cylinderical part (h_1) = 4 m

Slant height of conical part (l) = 40 m

and radius (r) = $\left(\frac{105}{2}\right) m$



surface area of the tent = curved surface area of conical part + curved surface area of cylinderical part

$= \pi rl + 2\pi rh$

$= \pi r(l + 2h) = \frac{22}{7} \times \frac{105}{2}(40 + 2 \times 4)\text{m}^2$

= 165 (40 + 8) = 165 × 48 m^2 = 7920 m^2

10. **(b)** $x^2 - 14x + 46 = 0$

Explanation: Given: $\alpha = 7 + \sqrt{3}$ and $\beta = 7 - \sqrt{3}$

$\therefore x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 (7 + \sqrt{3} + 7 - \sqrt{3})x + (7 + \sqrt{3})(7 - \sqrt{3}) = 0$

$\Rightarrow x^2 - 14x + (49 - 3) = 0$

$\Rightarrow x^2 - 14x + 46 = 0$

11. **(d)** Real and Distinct roots

Explanation: Comparing the given equation to the below equation

$ax^2 + bx + c = 0$

a = 1, b = -6, c = 6

$D = b^2 - 4ac$

D = (-6)2 - 4 × 1 × 6

D = 36 - 24

D = 12

D > 0.

If $b^2 - 4ac > 0$, then the equation has real and distinct roots

Hence Real and Distinct roots.

12. **(c)** 2

Explanation: $n^2 - n = n(n - 1)$. Since n and (n - 1) are consecutive integers. Therefore, one of them must be divisible by 2.

13. **(b)** $-y_1 : y_2$

Explanation: Let a point A on x-axis divides the line segment joining the points $P(x_1, y_1)$ $Q(x_2, y_2)$ in the ratio $m_1 : m_2$ and

let co-ordinates of A be (x, 0)

$\therefore 0 = \frac{m_1y_2+m_2y_1}{m_1+m_2} \Rightarrow 0 = m_1y_2 + m_2y_1$

$\Rightarrow m_1y_2 = -m_2y_1 \Rightarrow \frac{m_1}{m_2} = \frac{-y_1}{y_2}$

$\therefore$ Ratio is $-y_1 : y_2$

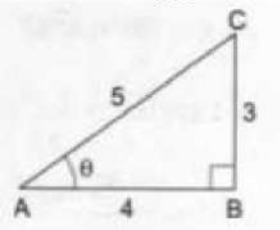
14. **(a)** $\frac{3}{4}$

Explanation: $\cos\theta = \frac{4}{5} = \frac{AB}{AC}$

$\therefore \quad BC^2 = AC^2 - AB^2 = 25 - 16 = 9$

$\Rightarrow$ BC = 3

$\therefore \tan\theta = \frac{BC}{AB} = \frac{3}{4}$



15. **(b)** 15

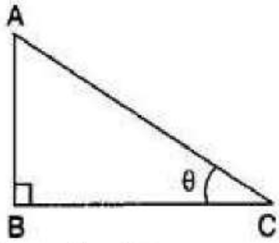
Explanation: Mode of 16, 15, 17, 16, 15, x, 19, 17, 14 is 15

$\because$ By definition mode of a number which has maximum frequency. Here, given that 15 is the mode i.e, 15 has maximum frequency

$\therefore$ x = 15

16. **(b)** 30°

Explanation:



Let Height of the pole AB = x m and length of the shadow BC = $\sqrt{3}x$ meters

$\therefore \tan\theta = \frac{\text{AB}}{\text{BC}}$

$\Rightarrow \tan\theta = \frac{x}{\sqrt{3}x}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan 30^\circ$

$\Rightarrow \theta = 30^\circ$

17. **(b)** parallel

Explanation: We have,

6x - 2y + 9 = 0

And, 3x - y + 12 = 0

Here, $a_1 = 6$, $b_1 = -2$ and $c_1 = 9$

$a_2 = 3$, $b_2 = -1$ and $c_2 = 12$

$\frac{a1}{a2} = \frac{6}{3} = \frac{2}{1}, \frac{b1}{b2} = \frac{-2}{-1} = \frac{2}{1}$ and $\frac{c_1}{c_2} = \frac{9}{12} = \frac{3}{4}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given system has no solution and the lines are parallel.

18. **(d)** 320

Explanation: Let the two numbers be x and y.

It is given that: $x \times y = 1600$

HCF = 5

We know, HCF $\times$ LCM = $x \times y$

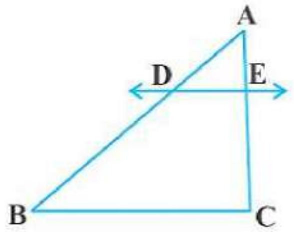
$\Rightarrow 5 \times \text{LCM} = 1600$

$\therefore \text{LCM} = \frac{1600}{5} = 320$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

We know that if a line is parallel to one side of a triangle then it divides the other two sides in the same ratio. This is the Basic Proportionality theorem. So, the Reason is correct.



By Basic Proportionality theorem, we have $\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{DB}{AD} = \frac{EC}{AE} \Rightarrow \frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$

$\Rightarrow \frac{DB+AD}{AD} = \frac{EC+AE}{AE} \Rightarrow \frac{AB}{AD} = \frac{AC}{AE} \quad \Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

So, the Assertion is correct.

20. **(c)** A is true but R is false.

Explanation: We have,

LCM (a, b) $\times$ HCF (a, b) = a $\times$ b

LCM $\times$ 5 = 150

LCM = $\frac{150}{5} = 30$

Section B

21. Number 50 p coins in the piggy bank = 100

Number of Re. 1 coins in the piggy bank = 50

Number of Rs. 2 coins in the piggy bank = 20

Number of Rs. 5 coins in the piggy bank = 10

$\therefore$ Total number of coins in the piggy bank = 100 + 50 + 20 -10 = 180

$\therefore$ Number of all possible outcomes = 180

i. Number of favourable outcomes to the event that the coin will be a 50 p coin = 100

$\therefore$ Probability that the coin will be a 50 p coin

$\frac{Number\ of\ favourable\ outcomes\ to\ the\ event\ that\ the\ coin\ will\ be\ a\ 50\ p\ coin}{Number\ of\ all\ possible\ outcomes}\ \frac{100}{180} = \frac{5}{9}$

ii. Number of favourable outcomes to the event that the coin will not be a Rs. 5 coin

= 100 + 50 + 20 = 170

$\therefore$ Probability that the coin will not be Rs. 5 coin

$\frac{Total\ no.\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes}\ \frac{170}{180} = \frac{17}{18}$

22. Let the polynomial be $ax^2 + bx + c$,

and is zeroes be α and β.

Then, $\alpha + \beta = 4 = -\frac{b}{a}$ and $\alpha\beta = 1 = \frac{c}{a}$

If a = 1, then b = -4 and c = 1.

So one quadratic polynomial which fits

the given conditions is $x^2 - 4x + 1$.

23. Let the ratio be K: 1

Coordinate of P are $\left(\frac{3K+2}{K+1}, \frac{7K-2}{K+1}\right)$

P lies on the line 2x + y - 4 = 0

$\Rightarrow 2\left(\frac{3K+2}{K+1}\right) + \frac{7K-2}{K+1} - \frac{4}{1} = 0$

$\Rightarrow$ 6K + 4 + 7K - 2 - 4K - 4 = 0

$\Rightarrow$ 9K - 2 = 0

$\Rightarrow K = \frac{2}{9}$ or 2 : 9

24. x - y = 8.................(1)

3 x - 3 y = 16.............(2)

Here, $a_1 = 1, b_1 = -1, c_1 = -8$

$a_2 = 3, b_2 = -3, c_2 = -16$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines represented by the equations(1) and (2) are parallel.

Therefore, equations (1) and (2) have no solution, i.e., the given pair of linear equation is inconsistent.

OR

Given pair of linear equations is

2x + 3y = 7

and 2px + py = 28 - qy

or 2px + (p + q)y - 28 = 0

On comparing with ax + by + c = 0 we get

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = - 7$;

And $a_2 = 2p$, $b_2 = (p + q)$, $c_2 = - 28$;

$\frac{a_1}{a_2} = \frac{2}{2p}$

$\frac{b_1}{b_2} = \frac{3}{p+q}$

$\frac{c_1}{c_2} = \frac{1}{4}$

Since, the pair of equations has infinitely many solutions i.e., both lines are coincident.

$a_1/a_2 = b_1/b_2 = c_1/c_2$

$\frac{1}{p} = \frac{3}{p+q} = \frac{1}{4}$

Taking first and third parts, we get

p = 4

Again, taking last two parts, we get

$\frac{3}{p+q} = \frac{1}{4}$

p + q = 12

Since p = 4

So, q = 8

Here, we see that the values of p = 4 and q = 8 satisfies all three parts.

Hence, the pair of equations has infinitely many solutions for all values of p = 4 and q = 8.

25. We know that the radius and tangent are perpendicular at their point of contact.

$\angle OPT = 90^\circ$

Now, $\angle$OPQ = $\angle$OPT - $\angle$TPQ

$= 90^\circ - 70^\circ = 20^\circ$

Since OP = OQ as both are radius

$\angle OPQ = \angle OQP = 20^\circ$ (Angle opposite to equal sides are equal)
Now, In isosceles triangle POQ,
$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$ (Angle sum property of a triangle)
$\angle POQ = 180^\circ - 20^\circ - 20^\circ = 140^\circ$

OR

Given: PQ and PR are tangents to a circle with centre O and $\angle QPR = 50^\circ$.
To find: $\angle QSR$
$\angle QOR + \angle QPR = 180^\circ$
$\Rightarrow \angle QOR + 50^\circ = 180^\circ$
$\Rightarrow \angle QOR = 130^\circ$
$\Rightarrow \angle QSR = \frac{1}{2} \angle QOR$
$\Rightarrow \angle QSR = \frac{1}{2} \times 130^\circ = 65^\circ$

Section C

26. 1. By Elimination method,
The given system of equation is :
3 x + 4 y = 10 ....(1)
2 x - 2 y = 2 ....(2)
Multiplying equation(2) by 2, we get
4 x - 4 y = 4 ....(2)
Adding equation (1) and equation (3), we get
7 x = 14
$\therefore \quad x = \frac{14}{7} = 2$
Substituting this value of x in equation (2), we get
2(2) - 2y = 2
$\Rightarrow \quad 4 - 2y = 2$
$\Rightarrow \quad 2y = 4 - 2$
$\Rightarrow \quad 2y = 2$
$\Rightarrow \quad y = \frac{2}{2} = 1$
So, the solution of the given system of equation is
x = 2, y = 1
2. By Substitution method,
The given system of equation is:
3 x + 4 y = 10.................(1)
2 x - 2 y = 2....................(2)
From equation(1)
$3x = 10 - 4y$
$x = \left(\frac{10-4y}{3}\right)$
Put value of x in equation (2),
$2x - 2y = 2$
$2\left(\frac{10-4y}{3}\right) - 2y = 2$
$\frac{2(10-4y)-2y(3)}{3} = 2$
$20 - 8y - 6y = 6$
$-14y = -14$
y = 1
Putting value of y = 1 in equation (2)
2x - 2 = 2
x = 2

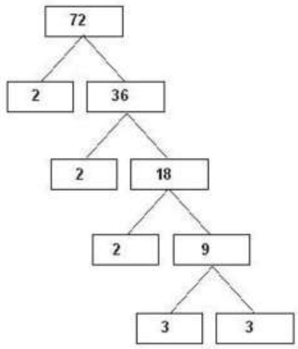
Therefore, x = 2, y = 1 is the solution.
Verification: Substituting x = 2, y = 1, we find that both the equation(1) and (2) are satisfied shown below:
$3x+4y=3(2)+4(1)=6+4=10$
$2x-2y=2(2)-2(1)=4-2=2$
Hence, the solution is correct.

27. In, ΔACB we have
$AB^2=AC^2+BC^2$
$\Rightarrow \quad AC=\sqrt{AB^2-BC^2}=\sqrt{29^2-21^2}=\sqrt{(29+21)(29-21)}=\sqrt{400}=20$ units
$\therefore \quad \sin\theta=\frac{AC}{AB}=\frac{20}{29}$ and $\cos\theta=\frac{BC}{AB}=\frac{21}{29}$
i. Using the values of $\sin\theta$ and, $\cos\theta$ we get
$\cos^2\theta+\sin^2\theta=\left(\frac{21}{29}\right)^2+\left(\frac{20}{29}\right)^2$
$=\frac{441+400}{841}=1$
ii. Using the values of $\sin\theta$ and, $\cos\theta$ we obtain
$\cos^2\theta-\sin^2\theta=\left(\frac{21}{29}\right)^2-\left(\frac{20}{29}\right)^2=\frac{21^2-20^2}{29^2}=\frac{(21+20)(21-20)}{841}=\frac{41}{841}$

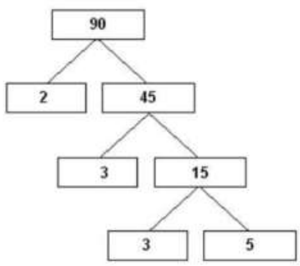
28. $\triangle ABC \sim \triangle DEF$ (Given)
Therefore $\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$
SO, $\frac{2x-1}{18}=\frac{2x+2}{3x+9}=\frac{3x}{6x}$
Now, taking $\frac{2x-1}{18}=\frac{3x}{6x}$ we have
$\frac{2x-1}{18}=\frac{1}{2}$
or 4 x - 2 = 18
or x = 5
Therefore, AB = 2 × 5 –1= 9, BC = 2 × 5 + 2 = 12,
CA = 3 × 5 = 15, DE = 18, EF = 3 × 5 + 9 = 24 and FD = 6 × 5 = 30
Hence, AB = 9 cm, BC = 12 cm, CA = 15 cm,
DE = 18 cm, EF = 24 cm and FD = 30 cm.

29. Let us prove $\sqrt{5}$ irrational by contradiction.
Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers *a* and b (b ≠ 0)
Such that $\sqrt{5}=\frac{a}{b}$
$\Rightarrow b\sqrt{5}=a$
Squaring both sides, we get
$\Rightarrow 5b^2=a^2$ ... (1)
It means that 5 is factor of a^2
Hence, 5 is also factor of *a* by Theorem. ... (2)
If, 5 is factor of *a* , it means that we can write *a* = 5*c* for some integer *c* .
Substituting value of *a* in (1) ,
$5b^2=25c^2$
$\Rightarrow b^2=5c^2$
It means that 5 is factor of b^2 .
Hence, 5 is also factor of *b* by Theorem. ... (3)
From (2) and (3) , we can say that 5 is factor of both *a* and *b* .
But, *a* and *b* are co-prime .
Therefore, our assumption was wrong. $\sqrt{5}$ cannot be rational. Hence, it is irrational.

OR



So, $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$



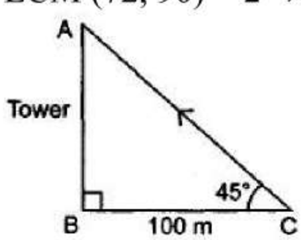
So, $90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$

Therefore,

HCF (72, 90) $= 2 \times 3^2 = 18$

LCM (72, 90) $= 2^3 \times 3^2 \times 5 = 360$

30.



$BC = 100\ m$

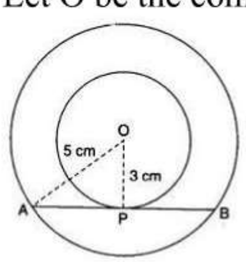
In right ABC,

$\frac{AB}{BC} = \tan 45^\circ$

$\Rightarrow \quad \frac{\mathbf{AB}}{100} = 1$

$\Rightarrow AB = 100m$

31. Let O be the common centre of the two concentric circles.



Let AB be a chord of the larger circle which touches the smaller circle at P.

Join OP and OA

Then, $\angle OPA = 90^o$ [$\because$The tangent at any point of a circle is perpendicular to th radius through the point of contact]

$\therefore OA^2 = OP^2 + AP^2$ .... By Pythagoras theorem

$\Rightarrow (5)^2 = (3)^2 + AP^2$

$\Rightarrow 25 = 9 + AP^2$

$\Rightarrow P^2 = 25 - 9$

$\Rightarrow AP^2 = 16$

$\Rightarrow$ AP = $\sqrt{16}$ = 4 cm

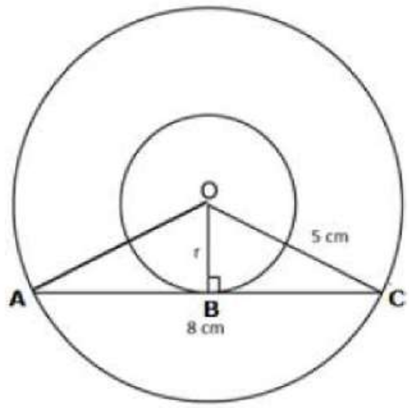
SInce the perpendicular from the centre of a circle to a chord bisects the chord, therfore,

AP = BP = 4 cm

$\therefore$ AB = AP + BP = AP + AP = 2AP = 2(4) = 8 cm

Hence, the required length is 8 cm.

OR



Since AC is a tangent to the inner circle.

$\angle OBC = 90^\circ$

AC is a chord of the outer circle.

We know that, the perpendicular drawn from the centre to a chord of a circle, bisects the chord.

AC = 2BC

8 = 2BC

$\Rightarrow$ BC = 4 cm

In ΔOBC,

By Pythagoras theorem,

$OC^2 = OB^2 + BC^2$

$\Rightarrow 5^2 = r^2 + 4^2$

$\Rightarrow r^2 = 5^2 - 4^2$

$\Rightarrow r^2 = 25 - 16$

$\Rightarrow r^2 = 9$ cm

$\Rightarrow$ r = 3 cm

Section D

32. Given:

ABCD is a trapezium,

Diagonals AC and BD are intersect at 0.

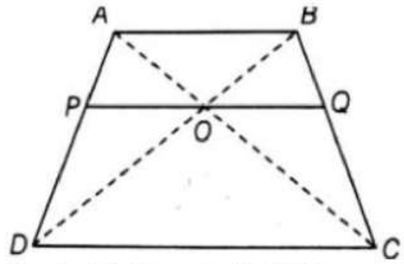
To prove: PQ || AB || DC.

PO = QO

Concepts Used:

AAA Similarity Criterion: If all three angles of a triangle equals to angles of another triangle, then both the triangles are similar.Basic Proportionality theorem: If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion

Proof:



In $\triangle$ABD and $\triangle$POD,

PO || AB ...[$\because$ PQ || AB]

$\angle$D = $\angle$D ...[common angle]

$\angle$ABD = $\angle$POD ...[corresponding angles]

$\therefore$ $\triangle$ABD ~ $\triangle$POD ...[by AAA similarity criterion]

Then,

OP/AB = PD/AD …(i) [by basic proportionality theorem]

In $\triangle$ABC and $\triangle$OQC,

OQ || AB ...[$\because$ OQ || AB]

$\angle$C = $\angle$C ...[common angle]

$\angle$BAC = $\angle$QOC ...[corresponding angle]

$\therefore$ $\triangle$ABC ~ $\triangle$OQC ...[by AAA similarity criterion]

Then,

OQ/AB = QC/BC …(ii) ...[by basic proportionality theorem]

Now, in $\triangle$ADC,

OP || DC

$\therefore$ AP/PD = OA/OC ...[by basic proportionality theorem] …(iii)

In $\triangle$ABC, OQ || AB

$\therefore$ BQ/QC = OA/OC ...[by basic proportionality theorem] …(iv)

From Equation (iii) and (iv),

AP/PD = BQ/QC

Adding 1 on both sides, we get,

= AP/PD + 1 = BQ/QC + 1

= ((AP + PD))/PD = (BQ + QC)/QC

= AD/PD = BC/QC

= PD/AD = QC/BC

= OP/AB = OQ/BC ...[from Equation (i) and (ii)]

$\Rightarrow$ OP/AB = OQ/AB ...[from Equation (iii)]

$\Rightarrow$ OP = OQ

Hence proved.

33. Given,

(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0

$\Rightarrow x^2 - ax - bx + ab + x^2 - bx - cx + bc + x^2 - cx - ax + ac = 0$

$\Rightarrow 3x^2 - 2ax - 2bx - 2cx + ab + bc + ca = 0$

For equal roots $B^2 - 4AC = 0$

or, $\{-2(a+b+c)\}^2 = 4 \times 3(ab+bc+ca)$

or, $4(a+b+c)^2 - 12(ab+bc+ca) = 0$

or, $a^2+b^2+c^2+2ab+2bc+2ac-3ab-3bc-3ac=0$

or, $\frac{1}{2}\left[2a^2+2b^2+2c^2-2ab-2ac-2bc\right]=0$

or, $\frac{1}{2}\left[\left(a^2+b^2-2ab\right)+\left(b^2+c^2-2bc\right)+\left(c^2+a^2-2ac\right)\right]=0$

or, $\frac{1}{2}\left[\left(a^2+b^2-2ab\right)+\left(b^2+c^2-2bc\right)+\left(c^2+a^2-2ac\right)\right]=0$

or, $(a-b)^2+(b-c)^2+(c-a)^2=0$ if $a \neq b \neq c$

Since $(a-b)^2>0,(b-c)^2>0(c-a)^2>0$

Hence, $(a-b)^2=0 \Rightarrow a=b$
$(a-c)^2=0 \Rightarrow b=c$
$(c-a)^2=0 \Rightarrow c=a$
$\therefore a=b=c$ Hence Proved.

OR

Since the given equation has equal roots,
$D=b^2-4ac=0$
Here, a = (3k + 1), b = 2(k + 1), c = 1
$[2(k+1)]^2-4(3k+1)(1)=0$
or, $4\left(k^2+2k+1\right)-(12k+4)=0$
or, $4k^2+8k+4-12k-4=0$
$\therefore 4k^2-4k=0$
k = 0,1
Put k = 0, in the given equation,
$x^2+2x+1=0$
or, $(x+1)^2=0$
or, x = -1
Again put k = 1, in the given equation,
$4x^2+4x+1=0$
$(2x+1)^2=0$
or, $x=-\frac{1}{2}$
Hence, roots = $-1, -\frac{1}{2}$

34. Mode:

Here, the maximum frequency is 23 and the class corresponding to this frequency is 35 - 45.
So, the modal class is 35 - 45.
Now, size (h) = 10
lower limit it (l) of modal class = 35
frequency (f_1) of the modal class = 23
frequency (f_0) of class previous the modal class = 21
frequency (f_2) of class succeeding the modal class = 14
$\therefore$ Mode = $l+\frac{f_1-f_0}{2f_1-f_0-f_2}\times h = 35+\frac{23-21}{2\times 23-21-14}\times 10$
$=35+\frac{2}{11}\times 10=35+\frac{20}{11}$
= 35 + 1.8 (approx.)
= 36.8 years (approx.)

Mean:-

Take a = 40, h = 10.

Age (in years)	Number of patients (f_i)	Class marks (x_i)	$d_i = x_i - 40$	$u_i=\frac{x_i-40}{10}$	f_iu_i
5-15	6	10	–30	–3	–18
15-25	11	20	–20	–2	–22
25-35	21	30	–10	–1	–21
35-45	23	40	0	0	0
45-55	14	50	10	1	14
55-65	5	60	20	2	10
Total	$\sum f_i=80$				$\sum f_iu_i$ = - 37

Using the step deviation method,

$\overline{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times$ h $= 40 + \left(\frac{-37}{80}\right) \times 10$

$= 40 - \frac{37}{8} = 40 - 4.63$

$= 35.37$ years

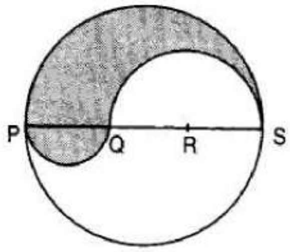
Interpretation:- Maximum number of patients admitted in the hospital are of the age 36.8 years (approx.), while on an average the age of a patient admitted to the hospital is 35.37 years.

35. PS = Diameter of a circle of radius 6 cm = 12 cm

$\therefore$ PQ = QR = RS = $\frac{12}{3}$ = 4cm, QS = QR + RS = (4 + 4)cm = 8 cm

Let P be the perimeter and A be the area of the shaded region.

P = Arc of semi-circle of radius 6 cm + Arc of semi-circle of radius 4 cm + Arc of semi-circle of radius 2 cm



$\Rightarrow P = (\pi \times 6 + \pi \times 4 + \pi \times 2)\text{cm} = 12\pi\text{cm}$

and, A = Area of semi-circle with PS as diameter + Area of semi-circle with PQ as diameter - Area of semi-circle with QS as diameter.

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7}\left(6^2 + 2^2 - 4^2\right) = \frac{1}{2} \times \frac{22}{7} \times 24 = \frac{264}{7}\text{cm}^2 = 37.71\ \text{cm}^2$

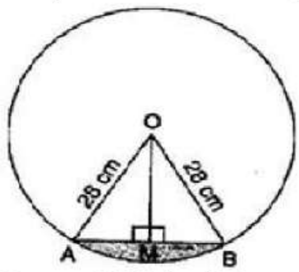
OR

r = 28 cm and $\theta = \frac{360}{6} = 60^\circ$

Area of minor sector = $\frac{\theta}{360}\pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 28 \times 28 = \frac{1232}{3}$

$= 410.67\ \text{cm}^2$

For, Area of $\triangle$AOB,



Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore \triangle$OMA $\cong$ OMB [RHS congruency]

$\therefore$ AM = BM [By CPCT]

$\Rightarrow$ AM = BM = $\frac{1}{2}$AB and $\angle$AOM = $\angle$BOM = $\frac{1}{2}$AOB = $\frac{1}{2} \times 60^\circ = 30^\circ$

In right angled triangle OMA, $\cos 30^\circ = \frac{OM}{OA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{28}$

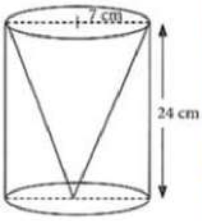
$\Rightarrow$ OM = $14\sqrt{3}$ cm

Also, $\sin 30° = \frac{AM}{OA}$
$\Rightarrow \frac{1}{2} = \frac{AM}{28}$
$\Rightarrow$ AM = 14 cm
$\Rightarrow$ 2 AM = 2 × 14 = 28 cm
$\Rightarrow$ AB = 28 cm
$\therefore$ Area of $\triangle$AOB = $\frac{1}{2}$ × AB × OM = $\frac{1}{2}$ × 28 × $14\sqrt{3}$ = 196 $\sqrt{3}$ = 196 × 1.7 = 333.2 cm
2
$\therefore$ Area of minor segment = Area of minor sector - Area of $\triangle$AOB
= 410.67 - 333.2 = 77.47 cm^2
$\therefore$ Area of one design = 77.47 cm^2
$\therefore$ Area of six designs = 77.47 × 6 = 464.82 cm^2
Cost of making designs = 464.82 × 0.35 = Rs. 162.68

Section E

36. **Read the text carefully and answer the questions:**

One day Vinod was going home from school, saw a carpenter working on wood. He found that he is carving out a cone of same height and same diameter from a cylinder. The height of the cylinder is 24 cm and base radius is 7 cm. While watching this, some questions came into Vinod's mind.



(i) Given height of cone = 24cm and radius of base = r = 7cm
Slant height of conical cavity,
$l = \sqrt{h^2 + r^2}$
$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625}$ = 25 cm

(ii) we know that r = 7cm, l = 25 cm
Curved surface area of conical cavity = πrl
$= \frac{22}{7} \times 7 \times 25 = 550$ cm^2

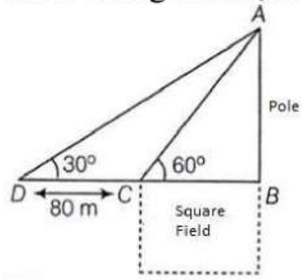
(iii) For cylinder height = h = 24cm, radius of base = r = 7cm
External curved surface area of cylinder
$= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 24 = 1056$ cm^2

OR

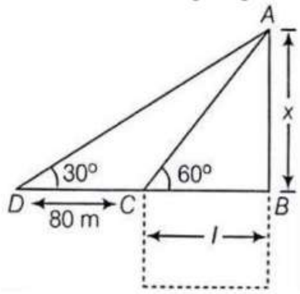
Curved surface area of conical cavity = πrl
$= \frac{22}{7} \times 7 \times 25 = 550$ cm^2
External curved surface area of cylinder
$= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 24 = 1056$ cm^2
$\frac{curved\ surface\ area\ of\ cone}{curved\ surface\ area\ of\ cylinder} = \frac{550}{1056} = \frac{275}{528}$
hence required ratio = 275:528

37. **Read the text carefully and answer the questions:**

Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite to this corner is 60°. When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30°.

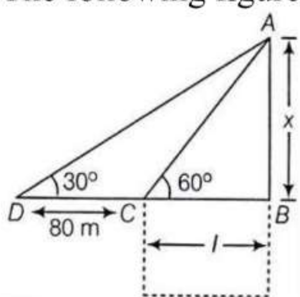


(i) The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.
Now, l = 40 metres
We get,
$x = \sqrt{3}\, l = 40\sqrt{3} = 69.28$
Thus, height of the pole is 69.28 metres.

(ii) The following figure can be drawn from the question:



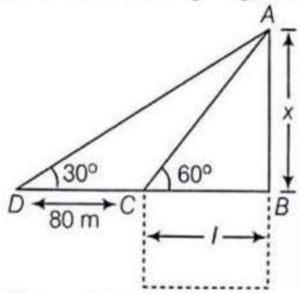
Here AB is the pole of height x metres and BC is one side of the square field of length l metres.
In $\triangle$ABC,
$\tan 60^{\circ} = \frac{x}{l}$
$\sqrt{3} = \frac{x}{l}$
$x = \sqrt{3}\, l$ ...(i)
Now, in $\triangle$ABD,
$\tan 30^{\circ} = \frac{x}{80+l}$
$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}l}{80+l}$ (From eq(i))
$80 + l = 3l$

$2l = 80$

$l = 40$

Thus, length of the field is 40 metres.

(iii)The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

Distance from Farmer at position C and top of the pole is AC.

In △ABC

$\cos 60^0 = \frac{CB}{AC}$

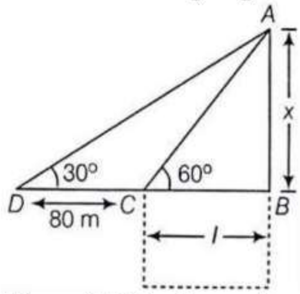
$\Rightarrow AC = \frac{CB}{\cos 60^0}$

$\Rightarrow AC = \frac{40}{\frac{1}{2}}$

$\Rightarrow$ AC = 80 m

OR

The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

Distance from Farmer at position D and top of the pole is AD

In △ABC

$\cos 30^0 = \frac{DB}{AD}$

$\Rightarrow AD = \frac{DB}{\cos 30^0}$

$\Rightarrow AD = \frac{120}{\frac{\sqrt{2}}{2}} = \frac{240}{\sqrt{3}}$

$\Rightarrow$ AC = 138.56 m

38. **Read the text carefully and answer the questions:**

Suman is celebrating his birthday. He invited his friends. He bought a packet of toffees/candies which contains 360 candies. He arranges the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.

(i) Let there be 'n' number of rows

Given 3, 5, 7... are in AP

First term a = 3 and common difference d = 2

$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow 360 = \frac{n}{2}[2 \times 3 + (n-1) \times 2]$
$\Rightarrow 360 = n[3 + (n - 1) \times 1]$
$\Rightarrow n^2 + 2n - 360 = 0$
$\Rightarrow (n + 20)(n - 18) = 0$
$\Rightarrow n = -20$ reject
$n = 18$ accept

(ii) Since there are 18 rows number of candies placed in last row (18^{th} row) is
$a_n = a + (n - 1)d$
$\Rightarrow a_{18} = 3 + (18 - 1)2$
$\Rightarrow a_{18} = 3 + 17 \times 2$
$\Rightarrow a_{18} = 37$

(iii) If there are 15 rows with same arrangement
$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow S_{15} = \frac{15}{2}[2 \times 3 + (15-1) \times 2]$
$\Rightarrow S_{15} = 15[3 + 14 \times 1]$
$\Rightarrow S_{15} = 255$
There are 255 candies in 15 rows.

OR

The number of candies in 12th row.
$a_n = a + (n - 1)d$
$\Rightarrow a_{12} = 3 + (12 - 1)2$
$\Rightarrow a_{12} = 3 + 11 \times 2$
$\Rightarrow a_{12} = 25$